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Transient boundary element algorithm for elastoplastic building floor slab analysis

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Abstract

A direct boundary element algorithm is developed for the dynamic analysis of thin elastoplastic building floor slabs directly supported by columns. The formulation employs the classical boundary element methodology dedicated to the analysis of elastoplastic plates. The method uses the static fundamental solution of the thin elastic plate problem. In this case, boundary as well as interior elements are used in the space descritization of the problem. This is due to the presence of plasticity and inertial effects in the integral formulation. An explicit time integration algorithm, employed on the incremental form of the matrix equation of motion, leads to the solution of the problem. Simple practical examples illustrate the accuracy and the efficiency of the proposed algorithm. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The introduction of the direct boundary element method (BEM) to the solution of the dynamic analysis problem of inelastic plates was first developed Fotiu et al. (1994), Providakis and Beskos (1993, 1994), Providakis et al. (1994) and Providakis (1999). As explained in the book on plates and shells edited by Beskos (1991) and the recent review article of Beskos (1995) there are basically two BEM approaches. The first BEM approach employs the elastodynamic fundamental solution of the problem in conjunction with modal synthesis and was applied to determine the dynamic response of viscoplastic damaging plates. The second one, named domain boundary element method (D/BEM), employs the elastostatic fundamental solution of the problem and was dedicated to determine the dynamic response of viscoplastic and elastoplastic plates. Following the above works, many other articles have already

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been published trying to extend the boundary element formulation to solve particular problems in engineering.

The boundary element algorithm proposed in this paper is applied to the dynamic analysis of elastoplastic building floor slabs. The importance of this analysis is approved by the increasing number of buildings designed with slabs directly supported by columns, basically due to the low cost of construction. The problem of the effect of internal supports in the elastic analysis of plates was successfully solved in the significant works of Bezine (1981), Hartmann and Zotemantel (1986) and particularly for the case of elastic building floor slab analysis in the work of Paiva and Ventourini (1985). Katsikadelis et al. (1988, 1990) were the first to apply the direct boundary element method to the dynamic analysis of elastic plates with internal supports. Their approach was mainly based on the capability to establish numerically Green's function for the corresponding static problem of the plate, subjected to the given boundary conditions without supports, using BEM.

In the present paper a D/BEM algorithm is presented to treat the time-dependent inelastic analysis of an elastoplastic building floor slab which besides the boundary supports, are also supported on points, lines or regions (patches) within the domain of the plate. It can be considered as an extension of the work of Providakis and Beskos (1993, 1994) and Providakis (1999) to include internal supports which may yield elastically, linearly or nonlinearly. The proposed algorithm can model a whole floor slab, with all the restrictions imposed by columns, employing very simple meshes and computes precisely the values of all bending and shear efforts including those at points located on the support areas or along load discontinuities. The Prandtl–Reuss stress–strain law based on Von Mises' yield condition are used to model hardening elastoplastic material behaviour. The descritized version of the equation of motion after using the boundary conditions are solved by the step-by-step time integration algorithm of the central predictor method.

Practical numerical examples presented in this paper evaluate the reliability of the proposed method and demonstrate its effectiveness.

2. Formulation of the problem

Consider a homogeneous, isotropic, thin floor slab of thickness h and of arbitrary domain S and boundary Γ, which is subjected to a transverse dynamic load under the Kirchoffs plate assumptions of small deformations. The equation of motion for the elastic plate bending can be reformulated in incremental form to include the plastic strain increments. Thus the incremental form of the governing equation of motion for the elastoplastic floor slab is

$$D\nabla^4 \delta w = \delta q - \rho h \delta \alpha - \nabla^2[\delta M^{\mathrm{p}}] \tag{1}$$

where δ denotes increments, ρ is the mass density of the slab, α is the acceleration of the lateral motion, q is the applied lateral load per unit area, $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the slab with E being the elastic modulus, ν, the Poisson ratio and the quantity $\nabla^2[\delta M^{\mathrm{p}}]$ describes the plastic moment effect and is given by

$$\nabla^2[\delta M^{\mathrm{p}}] = \frac{\partial^2 \delta M_x^{\mathrm{p}}}{\partial x^2} + 2\frac{\partial^2 \delta M_{xy}^{\mathrm{p}}}{\delta x \delta y} + \frac{\partial^2 \delta M_y^{\mathrm{p}}}{\partial y^2} \tag{2}$$

where xy indicates the middle plate plane. In the case of a floor slab resting on internal supports, the lateral load q is given by (Katsikadelis et al., 1988, 1990)

1. For a support at a point ξ_i:

$$q = -p\big[w(\xi_i)\big] + \bar{q} \quad \xi_i \in S \tag{3}$$

2. For a support on a line l_i:

$$q = -p\big[w(\xi)\big] + \bar{q} \quad \xi \in l_i \subset S \tag{4}$$

3. For a support on a region (patch) r_i:

$$q = -p\big[w(\xi)\big] + \bar{q} \quad \xi \in r_i \subseteq S \tag{5}$$

where $p = p(w)$ is, in general, a nonlinear function, describing the reacting forces at, say, interior point i and $\bar{q}$ is the dynamic lateral load applied on the plate.

Consequently, the differential equation of motion of an elastoplastic floor slab resting on boundary and/or internal supports in its incremental form is

$$D\nabla^4 \delta w = -p[\delta w] + \delta\bar{q} - \rho h \delta\alpha - \nabla^2[\delta M^{\mathrm{p}}] \tag{6}$$

The quantities $-p[\delta w]$, $\nabla^2[\delta M^{\mathrm{p}}]$ and $\rho h \delta\alpha$ simply appear in Eq. (6) as an additional effective lateral load. Thus, introducing the quantity δQ in the incremental form

$$\delta Q = -p[\delta w] + \delta\bar{q} - \rho h \delta\alpha \tag{7}$$

the equation of motion, Eq. (6) becomes

$$D\nabla^4 \delta w = \delta Q - \nabla^2[\delta M^{\mathrm{p}}] \tag{8}$$

3. Boundary integral equations

Considering Eq. (1), extending the work of Stern (1979) on plate elastostatics and following the procedure presented in Providakis and Beskos (1993, 1994) and Providakis (1999) for elastoplastic floor slab dynamics one can obtain for a point ξ, inside the region S of the slab, the integral equation

$$\begin{aligned}\delta w\left(\underset{\sim}{\xi}\right) = {} & \int_\Gamma \{U\delta V_n(\delta w) - \delta w V_n(U)\}\, \mathrm{d}\Gamma\left(\underset{\sim}{X}\right) + \int_\Gamma \left\{\frac{\partial}{\partial n}(\delta w) M_n(U) - \frac{\partial U}{\partial n}\delta M_n(\delta w)\right\} \mathrm{d}\Gamma\left(\underset{\sim}{X}\right) \\ & - \iint_S \left\{\frac{\partial^2 U}{\partial x^2}\delta M_x^{\mathrm{p}} + 2\frac{\partial^2 U}{\partial x^2}\delta M_{xy}^{\mathrm{p}} + \frac{\partial^2 U}{\partial y^2}\delta M_y^{\mathrm{p}} - U\delta Q\right\} \mathrm{d}S + \sum_{k=1}^{K}\{\|\delta w M_{nt}(U)\| \\ & - \|U\delta M_{nt}(\delta w)\|\}_k\end{aligned} \tag{9}$$

where n is the outward normal vector on Γ and δV_n, δM_n, $\frac{\partial}{\partial n}(\delta w)$ represent increments of equivalent shear force, normal bending moment and normal slope, respectively. The summed quantity denotes the discontinuity jump of the increment of the twisting moment δM_n at a corner on a Γ and the fundamental solution

$$U = \frac{r^2 \ln r}{8\pi D} \tag{10}$$

physically represents the lateral deflection at a point $\underset{\sim}{x}$ of an infinitely extended elastic plate due to a lateral concentrated unit load at ξ with x and ξ being two points in S and $r = |x - \xi|$. Explicit expressions for $\mathrm{d}U/\mathrm{d}n$, $M_n(U)$, $V_n(U)$ and $M_{nt}(U)$ can be found in Providakis and Beskos (1994). By bringing point ξ to a position Ξ on the boundary through a limiting process one can obtain from Eq. (9) the boundary integral equation

$$\frac{\Delta\omega}{2\pi}\delta w\left(\underset{\sim}{\Xi}\right) = \int_\Gamma \{U\delta V_n(\delta w) - \delta w V_n(U)\}\,\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) + \int_\Gamma \left\{\frac{\partial}{\partial n}(\delta w)M_n(U) - \frac{\partial U}{\partial n}\delta M_n(\delta w)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right)$$
$$- \int_S\int\left\{\frac{\partial^2 U}{\partial x^2}\delta M_x^{\mathrm{p}} + 2\frac{\partial^2 U}{\partial x^2}\delta M_{xy}^{\mathrm{p}} + \frac{\partial^2 U}{\partial y^2}\delta M_y^{\mathrm{p}} - U\delta Q\right\}\mathrm{d}S + \sum_{k=1}^{K}\{\|\delta w M_{nt}(U)\|$$
$$- \|U\delta M_{nt}(\delta w)\|\}_k \tag{11}$$

where the angle $\delta\omega = \omega_1 - \omega_2$ is the internal angle of the general corner boundary point Ξ in question with ω_1 and ω_2 being the angles between the tangent vectors on the left and right side of $\underset{\sim}{\Xi}$ and the axis x, respectively.

For a well posed plate bending problem one more boundary integral equation is needed. This is achieved by replacing U by the fundamental solution (Providakis and Beskos, 1994)

$$U_\varphi = \frac{\partial U}{\partial \zeta} = \frac{1}{8\pi D} r(1 + 2\ln r)\cos\varphi \tag{12}$$

which is actually the deflection caused by a couple rotating in the direction of a vector ζ and φ is the angle between r and the fixed direction ζ. Using U_φ one can finally produce for a general boundary point $\underset{\sim}{\Xi}$ the following integral equation

$$c_1\frac{\partial}{\partial n_1}\delta w\left(\underset{\sim}{\Xi}\right) + c_2\frac{\partial}{\partial n_2}\delta w\left(\underset{\sim}{\Xi}\right)$$
$$= \int_\Gamma\left\{U_\varphi\delta V_n(\delta w) - \left(\delta w - \delta w\left(\underset{\sim}{\Xi}\right)\right)V_n\left(U_\varphi\right)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right)$$
$$+ \int_\Gamma\left\{\frac{\partial}{\partial n}(\delta w)M_n\left(U_\varphi\right) - \frac{\partial U_\varphi}{\partial n}\delta M_n(\delta w)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right)$$
$$- \int\int_S\left\{\frac{\partial^2 U_\varphi}{\partial x^2}\delta M_x^{\mathrm{p}} + 2\frac{\partial^2 U_\varphi}{\partial x^2}\delta M_{xy}^{\mathrm{p}} + \frac{\partial^2 U_\varphi}{\partial y^2}\delta M_y^{\mathrm{p}}\right\}\mathrm{d}S$$
$$- \int\int_S\{-U_\varphi\delta\bar{q} + U_\varphi\rho h\delta\alpha\}\,\mathrm{d}S + \sum_{k=1}^{K}\left\{\|\delta w M_{nt}\left(U_\varphi\right)\| - \|U_\varphi\delta M_{nt}(\delta w)\|\right\}_k \tag{13}$$

where n_1, n_2 are the normal vectors on the left and right side of $\underset{\sim}{\Xi}$, respectively, and the characteristic functions c_1 and c_2 have the values

$$c_1 = \frac{\Delta\omega}{2\pi}\cos\theta + \frac{\nu}{2\pi}\left[\frac{1}{2}\sin 2\omega\cos\theta + \sin^2\omega\sin\theta\right]_{\omega_2}^{\omega_1}$$

$$c_2 = \frac{\Delta\omega}{2\pi}\sin\theta + \frac{\nu}{2\pi}\left[\sin^2\omega\cos\theta - \frac{1}{2}\sin^2\omega\sin\theta\right]_{\omega_2}^{\omega_1} \tag{14}$$

where θ is the angle between the global axis x and the unit normal vector n at the boundary point $\underset{\sim}{\Xi}$ in question. Explicit expressions for the kernel functions in Eq. (13) can be found in Providakis (1999).

By replacing the reactive forces on the internal supports by the load applied at each node of a mesh used to descritize the plate domain, Eqs. (9), (11) and (13) become

$$\begin{aligned}\delta w\left(\underset{\sim}{\xi}\right) = {} & \int_\Gamma \{U\delta V_n(\delta w) - \delta w V_n(U)\}\,\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) + \int_\Gamma \left\{\frac{\partial}{\partial n}(\delta w)M_n(U) - \frac{\partial U}{\partial n}\delta M_n(\delta w)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) \\ & - \int_S\int\left\{\frac{\partial^2 U}{\partial x^2}\delta M_x^{\mathrm{p}} + 2\frac{\partial^2 U}{\partial x^2}\delta M_{xy}^{\mathrm{p}} + \frac{\partial^2 U}{\partial y^2}\delta M_y^{\mathrm{p}} - U\delta\bar{q} + U\rho h\delta\alpha\right\}\mathrm{d}S - \sum_i Up\left[\delta w(\xi_i)\right] \\ & - \sum_i \int_{l_i} Up\left[\delta w(\xi)\right]\mathrm{d}S_\xi - \sum_i \int_{r_i}\int Up\left[\delta w(\xi)\right]\mathrm{d}S_\xi \\ & + \sum_{k=1}^{K}\{\|\delta w M_{nt}(U)\| - \|U\delta M_{nt}(\delta w)\|\}_k \end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\Delta\omega}{2\pi}\delta w\left(\underset{\sim}{\Xi}\right) = {} & \int_\Gamma \{U\delta V_n(\delta w) - \delta w V_n(U)\}\,\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) + \int_\Gamma \left\{\frac{\partial}{\partial n}(\delta w)M_n(U) - \frac{\partial U}{\partial n}\delta M_n(\delta w)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) \\ & - \int_S\int\left\{\frac{\partial^2 U}{\partial x^2}\delta M_x^{\mathrm{p}} + 2\frac{\partial^2 U}{\partial x^2}\delta M_{xy}^{\mathrm{p}} + \frac{\partial^2 U}{\partial y^2}\delta M_y^{\mathrm{p}}\right\}\mathrm{d}S - \int_S\int\{-U\delta\bar{q} + U\rho h\delta\alpha\}\,\mathrm{d}S \\ & - \sum_i Up\left[\delta w(\xi_i)\right] - \sum_i \int_{l_i} Up\left[\delta w(\xi)\right]\mathrm{d}S_\xi - \sum_i \int_{r_i}\int Up\left[\delta w(\xi)\right]\mathrm{d}S_\xi \\ & + \sum_{k=1}^{K}\{\|\delta w M_{nt}(U)\| - \|U\delta M_{nt}(\delta w)\|\}_k \end{aligned} \tag{16}$$

$$\begin{aligned} & c_1\frac{\partial}{\partial n_1}\delta w\left(\underset{\sim}{\Xi}\right) + c_2\frac{\partial}{\partial n_2}\delta w\left(\underset{\sim}{\Xi}\right) \\ & = \int_\Gamma\left\{U_\varphi\delta V_n(\delta w) - \left(\delta w - \delta w\left(\underset{\sim}{\Xi}\right)\right)V_n\left(U_\varphi\right)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) \\ & + \int_\Gamma\left\{\frac{\partial}{\partial n}(\delta w)M_n\left(U_\varphi\right) - \frac{\partial U_\varphi}{\partial n}\delta M_n(\delta w)\right\}\mathrm{d}\Gamma\left(\underset{\sim}{X}\right) \\ & - \int_S\int\left\{\frac{\partial^2 U_\varphi}{\partial x^2}\delta M_x^{\mathrm{p}} + 2\frac{\partial^2 U_\varphi}{\partial x^2}\delta M_{xy}^{\mathrm{p}} + \frac{\partial^2 U_\varphi}{\partial y^2}\delta M_y^{\mathrm{p}}\right\}\mathrm{d}S - \int\int\{-U_\varphi\delta\bar{q} + U_\varphi\rho h\delta\alpha\}\,\mathrm{d}S \\ & - \sum_i Up\left[\delta w(\xi_i)\right] - \sum_i \int_{l_i} Up\left[\delta w(\xi)\right]\mathrm{d}s_\xi - \sum_i \int_{r_i}\int Up\left[\delta w(\xi)\right]\mathrm{d}S_\xi \\ & + \sum_{k=1}^{K}\left\{\|\delta w M_{nt}\left(U_\varphi\right)\| - \|U_\varphi\delta M_{nt}(\delta w)\|\right\}_k \end{aligned} \tag{17}$$

To solve this new problem it is necessary to evaluate integrals of the form

$$\int_{l_i} Up\big[\delta w(\xi)\big]\,\mathrm{d}S_\xi, \quad \int_{l_i} U_\varphi p\big[\delta w(\xi)\big]\,\mathrm{d}s_\xi, \quad \int_{r_i}\int Up\big[\delta w(\xi)\big]\,\mathrm{d}S_\xi \quad \text{and} \quad \int_{r_i}\int U_\varphi p\big[\delta w(\xi)\big]\,\mathrm{d}S_\xi$$

4. Matrix formulations

A matrix formulation for the Eqs. (15)–(17) can be obtained by:

1. An interpolation of the boundary by piecewise polynomials (Hartmann and Zotemantel, 1986). An Hermittian interpolation is used for the increments of the deflections, δw, and the functions $\mathrm{d}U/\mathrm{d}n$, $M_n(U)$, $V_n(U)$ are approximated by a descritization of the boundary into a number of linear isoparametric elements.
2. A descritization of the domain S into a number of eight noded quadrilateral interior elements.

For Eqs. (16) and (17) and after the use of boundary conditions one obtains

$$[G_\Gamma]\{I\} + [H_\Gamma]\{Y\} + [Q_\Gamma]\{L\} + [F_\Gamma] = [J_\Gamma]\big\{p(W_\mathrm{S})\big\} + [M_\Gamma]\big\{\ddot{W}_\mathrm{I}\big\} \tag{18}$$

where $\{I\}$ and $\{Y\}$ are the vectors of the unknown and known increments of the nodal boundary values, $\{L\}$ is the vector of the known increments of the nodal load values and $\{F_\Gamma\}$ is the vector of the plastic moment terms, W_S is the vector of unknown increments of nodal lateral deflection at the supports nodes and $\ddot{W}_\mathrm{I}$ is the vector of the unknown increments of the nodal lateral accelerations at the inertia nodes.

In the same way, as Eq. (18), the descritized version of the integral Eq. (15) in matrix form after the use of boundary conditions reads

$$\{W\} = [G_\mathrm{S}]\{I\} + [H_\mathrm{S}]\{Y\} - [M_\mathrm{S}]\big\{\ddot{W}\big\} + [Q_\mathrm{S}]\{L\} + \{F_\mathrm{S}\} - [J_\mathrm{S}]\big\{p(W_\mathrm{S})\big\} \tag{19}$$

where the vector $W = \begin{bmatrix} W_\mathrm{S} \\ W_\mathrm{I} \end{bmatrix}$.

The elimination of vector $\{I\}$ between Eqs. (18) and (19) and the consideration of the fact that the values of the deflections W at the supports is fixed, e.g. $W = 0$, yields

$$\{W_\mathrm{I}\} + [M]^*\big\{\ddot{W}_\mathrm{I}\big\} = [Q]^*\{L\} + \{F\}^* + [H]^*\{Y\} \tag{20}$$

where

$$[M]^* = [M_\mathrm{S}] - [G_\mathrm{S}][G_\Gamma]^{-1}[M_\Gamma]$$

$$[Q]^* = [Q_\mathrm{S}] - [G_\mathrm{S}][G_\Gamma]^{-1}[Q_\Gamma]$$

$$[H]^* = [H_\mathrm{S}] - [G_\mathrm{S}][G_\Gamma]^{-1}[H_\Gamma]$$

$$\{F\}^* = \{F_\mathrm{S}\} - [G_\mathrm{S}][G_\Gamma]^{-1}\{F_\Gamma\}$$

$$[J]^* = [J_\mathrm{S}] - [G_\mathrm{S}][G_\Gamma]^{-1}[J_\Gamma] \tag{21}$$

The influence matrices [G], [H] and [J] can be considered as the sum of certain element matrices that describe the influence of the boundary element layers and interior support terms. All the boundary integrals in (15)–(17) are singular and they must be understood in the sense of the Cauchy principal value and can be evaluated according to the procedure presented in Providakis (1999).

5. Stress–strain relations

After some manipulations established in Providakis and Beskos (1993, 1994) the plastic strains can be given by the matrix equation

$$\delta\{\varepsilon\}^{\mathrm{p}} = [D]^{*}\delta\{\varepsilon\} \tag{22}$$

where $[D]^{*} = [I] - [D]^{\mathrm{e}^{-1}}[D]^{\mathrm{ep}}$ with [I] being the identity matrix, $[D]^{\mathrm{e}}$ is the elasticity matrix and

$$[D]^{\mathrm{ep}} = [D]^{\mathrm{e}} - [D]^{\mathrm{e}}\left\{\frac{\partial F}{\partial\{\sigma\}}\right\}\left\{\frac{\partial F}{\partial\{\sigma\}}\right\}^{\mathrm{T}}[D]^{\mathrm{e}}\left(H' + \left\{\frac{\partial F}{\partial\{\sigma\}}\right\}^{\mathrm{T}}[D]^{\mathrm{e}}\left\{\frac{\partial F}{\partial\{\sigma\}}\right\}\right)^{-1} \tag{23}$$

with H' being the slope of the uniaxial effective stress versus plastic strain curve. The Von Mises' yield surface for the present case is given by the equation

$$F = \left[\sigma_{xx}^{2} + \sigma_{yy}^{2} - \sigma_{xx}\sigma_{yy} + 3\sigma_{xy}^{2}\right]^{1/2} - \bar{\sigma} \tag{24}$$

where $\bar{\sigma}$ is the uniaxial effective stress.

6. Solution strategy

The values of the nodal lateral deflections w_i at every time station are obtained by integrating forward in time Eq. (20) through an explicit central difference predictor algorithm. Denoting by $\underline{w}$, $\underline{\dot{w}}$ and $\underline{\ddot{w}}$ the total lateral deflection, velocity and acceleration vectors and by $\delta\underline{w}$, $\delta\underline{\ddot{w}}$ and $\delta\underline{\ddot{w}}$ their corresponding increments, the process of numerical time integration is carried out as follows:

Step 1. The initial distribution of lateral deflections, velocities and accelerations are prescribed, e.g.,

$$\underline{w_0},\ \underline{\dot{w}_0} \quad \text{and} \quad \underline{\ddot{w}_0} = 0 \tag{25}$$

where the subscript 0 denotes the time instant $t = 0$.

Step 2. Lateral deflections at the end of the time instant $t = \Delta t$ are calculated from the equation

$$\underline{w_1} = \underline{w_0} + \underline{\dot{w}_0}\Delta t + \frac{1}{2}\underline{\dot{w}_0}(\Delta t)^2 \tag{26}$$

Step 3. The incremental lateral deflections at the time instant $t = i\Delta t$ are computed as

$$\delta\underline{w_i} = \underline{w_i} - \underline{w_{i-1}} \tag{27}$$

and thus the increments δw_i become known.

Step 4. The second derivatives of the lateral deflections increments δw_i can be computed by using the derivatives of the rotations

$$\delta\varphi_x = \frac{\partial \delta w}{\partial x}, \qquad \delta\varphi_y = \frac{\partial \delta w}{\partial y} \tag{28}$$

which, in turn, can be resulted at the time instant $t = i\Delta t$ by using the finite difference method. For example, derivatives of the rotations $\delta\varphi_x$ and $\delta\varphi_y$ at the point (x_k, y_k) can be obtained from the relation

$$\left\{ \begin{matrix} \dfrac{\partial \delta\varphi_x}{\partial x}, \dfrac{\partial \delta\varphi_y}{\partial x} \\ \dfrac{\partial \delta\varphi_x}{\partial y}, \dfrac{\partial \delta\varphi_y}{\partial y} \end{matrix} \right\}_{\text{at } x_k,\, y_k} \approx \left\{ \begin{matrix} \dfrac{\{\delta\varphi_x, \delta\varphi_y\} \text{ at } (x_k + \Delta x, y_k) - \{\delta\varphi_x, \delta\varphi_y\} \text{ at } (x_k - \Delta x, y_k)}{2\Delta x} \\ \dfrac{\{\delta\varphi_x, \delta\varphi_y\} \text{ at } (x_k, y_k + \Delta y) - \{\delta\varphi_x, \delta\varphi_y\} \text{ at } (x_k, y_k - \Delta y)}{2\Delta y} \end{matrix} \right\} \tag{29}$$

i.e., by using their previous approximated values at the four different points $(x_k + \Delta x, y_k)$, $(x_k - \Delta x, y_k)$, $(x_k, y_k + \Delta y)$, $(x_k, y_k - \Delta y)$. Thus, the vector $\delta\{\varepsilon\}$ can be evaluated from the following equations

$$\delta\varepsilon_x = \frac{\partial \delta\varphi_x}{\partial x}$$

$$\delta\varepsilon_y = \frac{\partial \delta\varphi_y}{\partial y}$$

$$\delta\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial \delta\varphi_x}{\partial y} + \frac{\partial \delta\varphi_y}{\partial x} \right) \tag{30}$$

Step 5. The increment of the plastic strain vector $\delta\{\varepsilon\}^{\mathrm{p}}$ at the time instant $t = i\Delta t$ can be obtained from Eq. (22)

Step 6. The increments of the plastic moments δM_x^{p}, δM_y^{p} and $\delta M_{xy}^{\mathrm{p}}$ at the time instant $t = i\Delta t$ are then computed from the relations

$$\delta M_x^{\mathrm{p}} = D\left[\frac{\partial \delta\varphi_x^{\mathrm{p}}}{\partial x} + \nu \frac{\partial \delta\varphi_y^{\mathrm{p}}}{\partial y} \right]$$

$$\delta M_y^{\mathrm{p}} = D\left[\frac{\partial \delta\varphi_y^{\mathrm{p}}}{\partial y} + \nu \frac{\partial \delta\varphi_x^{\mathrm{p}}}{\partial x} \right]$$

$$\delta M_{xy}^{\mathrm{p}} = D\left[\frac{\partial \delta\varphi_x^{\mathrm{p}}}{\partial y} + \frac{\partial \delta\varphi_y^{\mathrm{p}}}{\partial x} \right] \tag{31}$$

Step 7. The increments of the moments δM_x, δM_y and δM_{xy} as well as the shear forces δQ_x and δQ_y at the time instant $t = i\Delta t$ are evaluated from the relations

$$\delta M_x = D\left[\frac{\partial \delta\varphi_x}{\partial x} + v\frac{\partial \delta\varphi_y}{\partial y}\right] - \delta M_x^{\mathrm{p}}$$

$$\delta M_y = D\left[\frac{\partial \delta\varphi_y}{\partial y} + v\frac{\partial \delta\varphi_x}{\partial x}\right] - \delta M_y^{\mathrm{p}}$$

$$\delta M_{xy} = \frac{D(1-v)}{2}\left[\frac{\partial \delta\varphi_x}{\partial y} + \frac{\partial \delta\varphi_y}{\partial x}\right] - \delta M_{xy}^{\mathrm{p}} \tag{32}$$

Step 8. The increments of the total stress vector $\delta\{\sigma\} = \delta\{\sigma_x, \sigma_y, \tau_{xy}\}$ at $t = i\Delta t$ is obtained by using the expression

$$\delta\{\sigma_x, \sigma_y, \tau_{xy}\} = \frac{12\tilde{z}}{h^3}\delta\{M_x, M_y, M_{xy}\} \tag{33}$$

Step 9. The values of stresses, moments and shear forces at $t = i\Delta t$ are obtained by using the relation

$$Z_i = Z_{i-1} + \delta Z \tag{34}$$

where Z stands for stresses, moments or shear forces.

Step 10. The yield surface F is computed in accordance with V. Mises yield criterion and the yielding is checked.

Step 11. The incremental acceleration is evaluated in a nodewise manner for the time instant $t = i\Delta t$ by solving Eq. (20) for $\delta\{\ddot{W}\}$.

Step 12. The total acceleration at $t = i\Delta t$ is computed from

$$\underline{\ddot{w}_i} = \underline{\ddot{w}_{i-1}} + \delta\underline{\ddot{w}_i} \tag{35}$$

Step 13. The total and incremental generalized displacements at time $t = (i+1)\Delta t$ are computed from

$$\underline{w_{i+1}} = 2\underline{w_i} - \underline{w_{i-1}} + \underline{\ddot{w}_i}(\Delta t)^2 \tag{36}$$

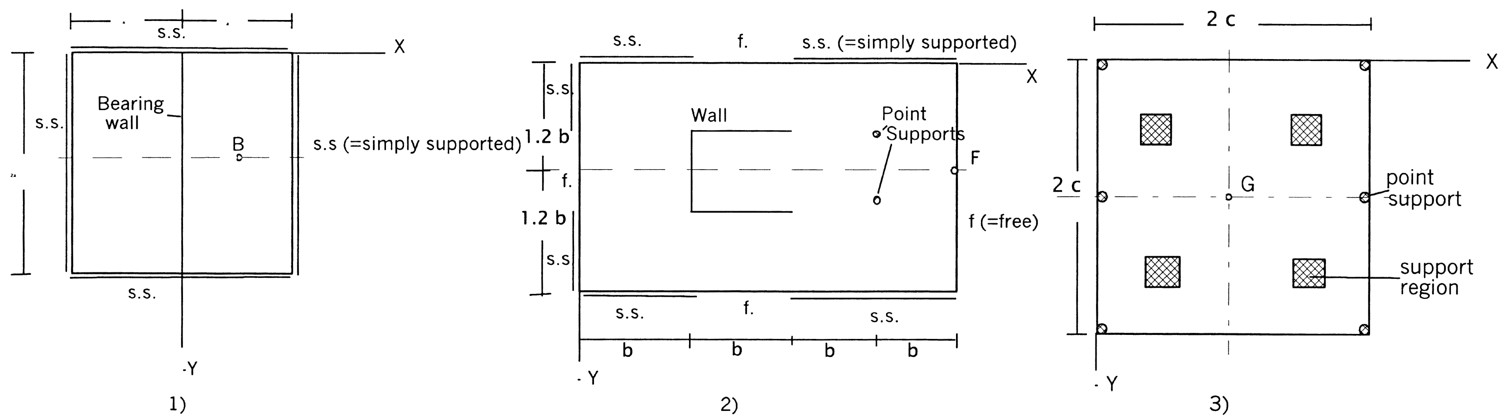
$$\delta\underline{w_{i+1}} = \underline{w_{i+1}} - \underline{w_i} \tag{37}$$

Step 14. Repetition of Steps 4–13 for each time increment Δt.

It should be noted here, that the above scheme becomes unstable when Δt exceeds a certain critical value, so that, an estimation of the critical time step is crucial for the subsequent analysis. In the present study the estimation of Tsui and Tong (1971) with minor modifications is found to be very well suited to obtain a reliable description of the critical time step length. This estimation can be written as

$$\Delta t = 0.70L_{\mathrm{s}}\left\{\frac{\rho\left(1+v^2\right)/E}{2+0.83(1-v)\left[1+1.5(L_s/t)^2\right]}\right\}^{1/2} \tag{38}$$

where L_{s} is the smallest distance between adjacent nodes of any parabolic element used.



Figs. 1–3. Interior element mesh for: 1) Square, simply supported two-span continuous plate. 2) Rectangular plate with mixed boundary conditions and complicated internal support. 3) Square plate with mixed boundary conditions and internally supported on square regions.

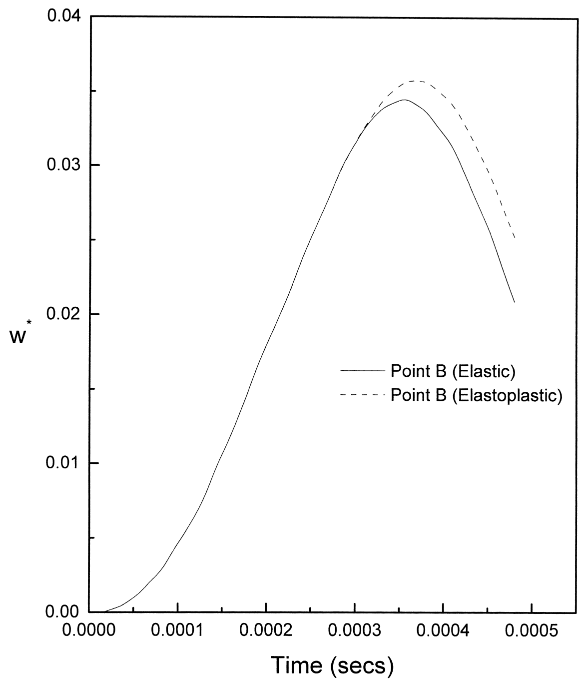


Fig. 4. Dynamic elastic and elastoplastic deflection history $W^*(t) = w(t)/(qa^4/D)$ at point B of the plate of Fig. 1.

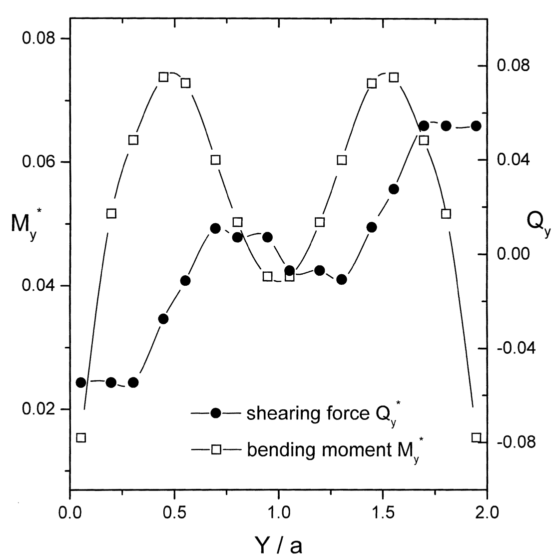


Fig. 5. Bending moment $M^* = M_y/qa^2$ and shearing force $Q^* = Q_y/qa$ variations along the line $x = a/2$ of the plate of Fig. 1 at $t = 0.0002$ s.

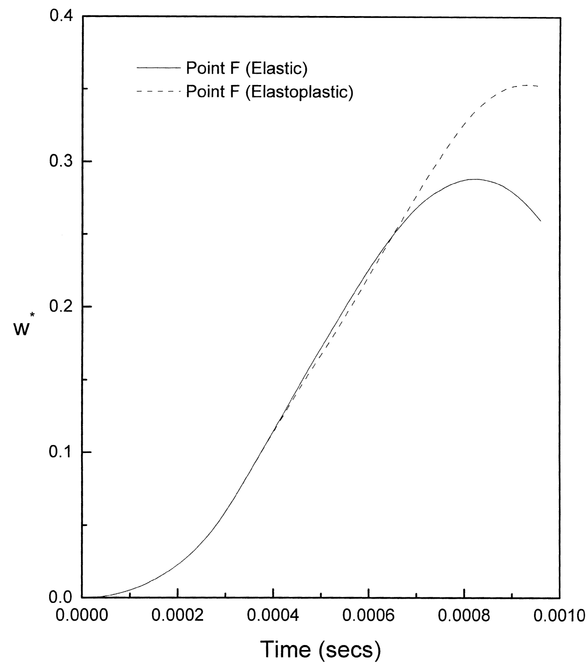


Fig. 6. Dynamic elastic and elastoplastic deflection history $W^*(t) = w(t)/(qa^4/D)$ at point F of the plate of Fig. 2.

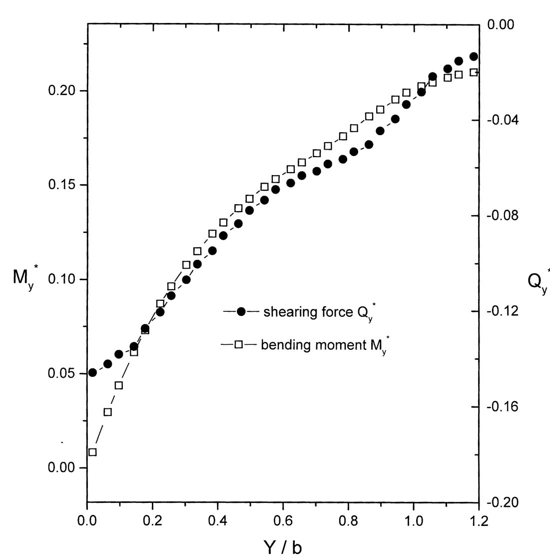


Fig. 7. Bending moment $M^* = M_y/qa^2$ and shearing force $Q^* = Q_y/qa$ variations along the line $x = 4b$ of the plate of Fig. 2 at $t = 0.0004$ s.

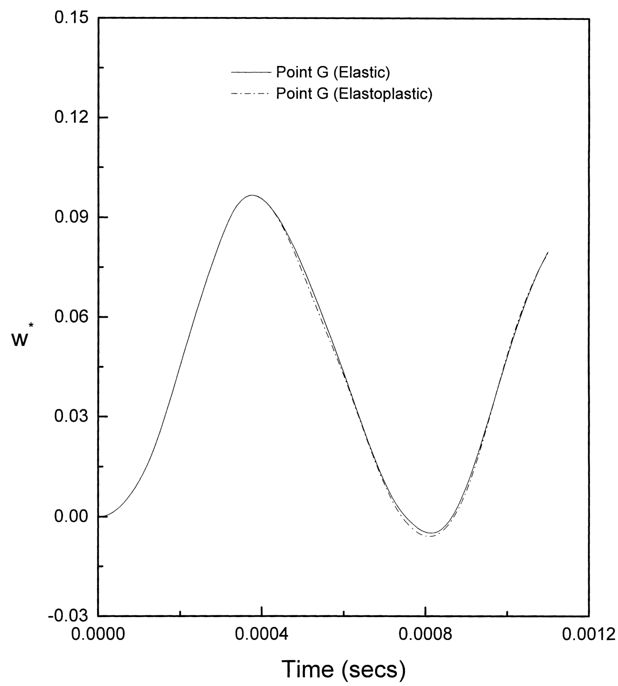


Fig. 8. Dynamic elastic and elastoplastic deflection history $W^*(t) = w(t)/(qa^4/D)$ at point G of the plate of Fig. 3.

7. Numerical examples

To illustrate the accuracy of the proposed method a computer program based on the analysis presented in the previous sections has been written. Three numerical examples of elastoplastic floor slabs with different boundary and interior conditions subjected to impulsive load have been studied (Figs. 1–3).

7.1. Example 1

Consider a square simply supported floor slab resting on a line support along the midspan and subjected to a uniformly distributed suddenly applied load (Fig. 1). Fig. 4 shows the dynamic elastic and elastoplastic response of the point B of the slab. Fig. 5 depicts the variation of the bending moment $M^* = M_y/qa^2$ and shearing force $Q^* = Q_y/qa$ along the line $x = a/2$ as computed by the present computer algorithm at $t = 0.0002$ s.

7.2. Example 2

As for the second example, a rectangular floor slab with mixed boundary conditions and complicated internal supports is considered which is subjected to a suddenly applied uniform load (Fig. 2). In Fig. 6 elastic and elastoplastic time variation of the deflection at the point F of the slab is shown. Fig. 7 shows the variation of the bending moment $M^* = M_y/qa^2$ and shearing force $Q^* = Q_y/qa$ along the line $x = 4b$ at $t = 0.0004$ s.

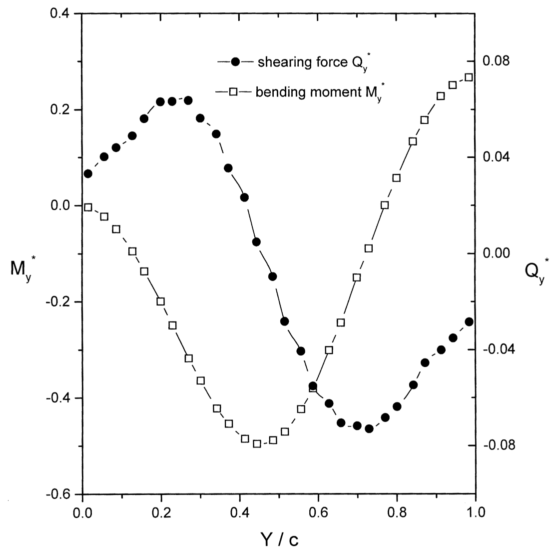


Fig. 9. Bending moment $M^* = M_y/qa^2$ and shearing force $Q^* = Q_y/qa$ variations along the line $x = c$ at $t = 0.0005$ s.

7.3. Example 3

In this example, a square floor slab with mixed boundary conditions and supported on four symmetrically located interior square regions (patches) has been considered. The sides of the interior regions has been taken equal to the seventh part of the whole plate side (Fig. 3). The computed response of the points G is depicted in Fig. 8. Fig. 9 shows the variation of the bending moment $M^* = M_y/qa^2$ and shearing force $Q^* = Q_y/qa$ along the line $x = c$ at $t = 0.0005$ s.

8. Conclusions

In this paper a domain/boundary element method has been presented for solving dynamic elastoplastic plate problems which, in addition to the boundary supports, are also supported inside the domain on points (columns), lines (walls) and regions (patches). On the basis of the preceding developments the following conclusions can be deduced:

(a) The BEM solution is very well suited to the dynamic elastoplastic problem of thin plates resting on internal supports.
(b) Plates having an arbitrary shape and supported to all kinds of boundary conditions and loading can be effectively analyzed.
(c) The examples analyzed in this paper emphasize the accuracy of the boundary element formulation.
(d) Besides the capability of the proposed method to treat linear elastic yielding of internal supports it can also be used to solve nonlinear elastic ones.

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